**Lecture10.**

**Rolle’s Theorem. Lagrange’s theorem. Increasing and decreasing Functions. Fermat’s theorem. Cauchy’s Theorem.**

**Mean-value theorems**

1. **Rolle’s Theorem.**

**a)**If a function $f(x)$ is continuous on the closed interval $a\leq x\leq b$,

**b)** has a derivative $f'(x)$ at every interior point of this interval,

**c)** and $f\left(a\right)=f\left(b\right),$

then the argument *x* has at least one value , where  such that

 (1)

1. **Lagrange’s Theorem.**

**a)** If a function $f(x)$ is continuous on the closed interval $a\leq x\leq b$,

**b)** has a derivative $f'(x)$ at every interior point of this interval, then

 (2)

where 

**Increasing and decreasing Functions.**

**Theorem.** Let J be an open interval, and let I be an interval consisting of all the points of J and possibly one or both of the endpoints of J. Suppose that f is continuous on I and differentiable on J.

1. If $f^{'}\left(x\right)>0$ for all *x* in J, then *f* is increasing on I,
2. If $f^{'}\left(x\right)<0$ for all x in J, then f is decreasing on I,
3. If $f^{'}\left(x\right)\geq 0$ for all x in J, then f is nondecreasing on I.
4. If $f^{'}\left(x\right)\leq 0$ for all x in J, then f is nonincreasing on I.
5. **Cauchy’s Theorem.**

**a)** If functions $f(x)$ and $g(x)$ are continuous on the closed interval $a\leq x\leq b$,

**b)** and for $a<x<b$ have a derivatives that do not vanish simultaneously, c) and $g\left(a\right)\ne g\left(b\right),$ then

 (3)

where 

1. **Fermat’s Theorem.** If f is defined on an open interval *(a,b)* and achieves a maximum (orminimum) value at the point *c* in *(a,b),* and if $f'(x)$ exists, then

$$f^{'}\left(c\right)=0$$

( values of *x* where $f^{'}\left(x\right)=0$ are called critical points of the function *f*).